K*riptografi* ***A****tasi* ***Z****arah* Digital Signature (KAZ-SIGN)

Algorithm Specifications and Supporting Documentation

(Version 1.6.1)

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1. INTRODUCTION

The proposed KAZ Digital Signature scheme, KAZ-SIGN (in Malay *Kriptografi Atasi Zarah* - translated literally “cryptographic techniques overcoming particles”; particles here referring to the photons) is built upon the hard mathematical problem coined as the Modular Reduction Problem (MRP). The idea revolves around the difficulty of reconstructing an unknown parameter from a given modular reducted value of that parameter. The target of the KAZ-SIGN design is to be a quantum resistant digital signature candidate with short verification keys and signatures, verifying correctly approximately 100% of the time, based on simple mathematics, having fast execution time and a potential candidate for seamless drop-in replacement in current cryptographic software and hardware ecosystems.

2. THE DESIGN IDEALISME

(i) To be based upon a problem that could be proven analytically to require exponential time to be solved;

(ii) To be able to prove analytically that the cryptosystem is indeed resistant towards quantum computers;

(iii) To utilize problems mentioned in point (i) above in its full spectrum without having to induce “weaknesses” in order for a trapdoor to be constructed;

(iv) To use “simple” mathematics in order to achieve maximum simplicity in design, such that even practitioners with limited mathematical background will be able to understand the arithmetic;

(v) Achieve 128 and 256-bit security with key length roughly equivalent to the non-quantum secure Elliptic Curve Cryptosystem (ECC);

(vi) To achieve maximum speed upon having simplicity in design and short key length;

(vii) To have a sufficiently large signature space;

(viii) The computation overhead for both signing and verification increases slightly even if the key size increases in the future;

(ix) To be able to be mounted on hardware with ease;

(x) The plaintext to signature expansion ratio is kept to a minimum.

One of our key strategy to obtain items (i) - (v) was by utilizing our defined Modular Reduction Problem (MRP). It is defined in the following section.

3. MODULAR REDUCTION PROBLEM (MRP)

Let $N = \prod_{i=1}^{j} p_i$ be a composite number and $n = \ell(N)$. Let p_k be a factor of N. Choose $\alpha \in (2^{n-1}, N)$. Compute $A \equiv \alpha \pmod{p_k}$.

The MRP is, upon given the values (A, N, p_k), one is tasked to determine $\alpha \in (2^{n-1}, N)$.

4. COMPLEXITY OF SOLVING THE MRP

Let $n_{p_k} = \ell(p_k)$ be the bit length of p_k. The complexity to obtain α is $O(2^{n-n_{p_k}})$. When deploying Grover's algorithm on a quantum computer, the complexity to obtain α is $O(2^{\frac{n-n_{p_k}}{2}})$. In other words, if $p_k \approx N^{\delta}$, for some $\delta \in (0,1)$, the complexity to obtain α is $O(N^{1-\delta})$. When deploying Grover's algorithm on a quantum computer, the complexity to obtain α is $O(N^{\frac{1-\delta}{2}})$.

5. THE HIDDEN NUMBER PROBLEM (HNP) (Boneh and Venkatesan, 2001)

Fix p and u. Let $O_{\alpha,g}(x)$ be an oracle that upon input x computes the most u significant bits of $\alpha g^x \pmod{p}$. The task is to compute the hidden number $\alpha \pmod{p}$ in expected polynomial time when one is given access to the oracle $O_{\alpha,g}(x)$. Clearly, one wishes to solve the problem with as small u as possible. Boneh and Venkatesan (2001) demonstrated that a bounded number of most significant bits of a shared secret are as hard to compute as the entire secret itself.

The initial idea of introducing the HNP is to show that finding the u most significant bits of the shared key in the Diffie-Hellman key exchange using users public key is equivalent to computing the entire shared secret key itself.

6. THE HERMANN MAY REMARKS (Herrmann and May, 2008)

We will now observe two remarks by Herrmann and May. It discusses the ability and inability to retrieve variables from a given modular multivariate linear equation. But before that we will put forward a famous theorem of Minkowski that relates the length of the shortest vector in a lattice to the determinant (see Hoffstein et al. (2008)).

Theorem 1. *In an ω-dimensional lattice, there exists a non-zero vector v with*

$$\|v\| \leq \sqrt{\omega}\, det(L)^{\frac{1}{\omega}}$$

In lattices with fixed dimension we can efficiently find a shortest vector, but for arbitrary dimensions, the problem of computing a shortest vector is known to be NP-hard under randomized reductions (see Ajtai (1998)). The LLL algorithm, however, computes in polynomial time an approximation of the shortest vector, which is sufficient for many applications.

Remark 1. *Let $f(x_1, x_2, \ldots, x_k) = a_1x_1 + a_2x_2 + \ldots + a_kx_k$ be a linear polynomial. One can hope to solve the modular linear equation $f(x_1, x_2, \ldots, x_k) \equiv 0 \pmod{N}$, that is to be able to find the set of solutions $(y_1, y_2, \ldots, y_k) \in \mathbb{Z}_N^k$, when the product of the unknowns are smaller than the modulus. More precisely, let X_i be upper bounds such that $|y_i| \leq X_i$ for $1, \ldots, k$. Then one can roughly expect a unique solution whenever the condition $\prod_i X_i \leq N$ holds (see Herrmann and May (2008)). It is common knowledge that under the same condition $\prod_i X_i \leq N$ the unique solution $(y_1, y_2, \ldots, y_k)$ can heuristically be recovered by computing the shortest vector in an k-dimensional lattice by the LLL algorithm. In fact, this approach lies at the heart of many cryptographic results (see Bleichenbacher and May (2006); Girault et al. (1990) and Nguyen (2004)).*

We would like to provide the reader with the conjecture and remark given in Herrmann and May (2008).

Conjecture 1. *If in turn we have $\prod_i X_i \geq N^{1+\varepsilon}$ then the linear equation $f(x_1, x_2, \ldots, x_k) = \sum_{i=1}^k a_ix_i \equiv 0 \pmod{N}$ usually has N^ε many solutions, which is exponential in the bit-size of N.*

Remark 2. *From Conjecture 1, there is hardly a chance to find efficient algorithms that in general improve on this bound, since one cannot even output all roots in polynomial time.*

7. THE KAZ-SIGN DIGITAL SIGNATURE ALGORITHM

7.1 Background

This section discusses the construction of the KAZ-SIGN scheme. We provide information regarding the key generation, signing and verification procedures. But first, we will put forward functions that we will utilize and the system parameters for all users.

7.2 Utilized Functions

Let $H(\cdot)$ be a hash function. Let $\phi(\cdot)$ be the usual Euler-totient function. Let $\ell(\cdot)$ be the function that outputs the bit length of a given input. Let $CRT([F,G],[P,Q])$ be the Chinese Remainder Theorem upon F modulo P and G modulo Q where P and Q are co-prime.

7.3 System Parameters

From the given security parameter k, determine parameter j. Next generate a list of the first j-primes larger than 2, $P = \{p_i\}_{i=1}^j$. Let $N = \prod_{i=1}^j p_i$. As an example, if $j = 43$, N is 256-bits. Let $n = \ell(N)$ be the bit length of N. Choose a random prime in $g \in \mathbb{Z}_N$ of order G_g where at most $G_g \approx N^\delta$ for a chosen value of $\delta \in (0,1)$ and $\delta \to 0$. That is,

$g^{G_g} \equiv 1 \pmod N$. Choose a random prime $R \in \mathbb{Z}_{\phi(N)}$ of order G_R, where $G_R \approx \phi(N)^{\varepsilon}$ for $\varepsilon \to 1$. That is, choose R with a large order in $\mathbb{Z}_{\phi(N)}$. Let $n_{G_R} = \ell(G_R)$ be the bit length of G_R. Such R, has its own natural order in $Z_{\phi(G_g)}$. Let that order be denoted as G_{Rg}. We can observe the natural relation given by $R^{G_{Rg}} \equiv 1 \pmod{G_g}$ where $\phi(N) \equiv 0 \pmod{G_g}$ and $\phi(G_g) \equiv 0 \pmod{G_{Rg}}$. Let $n_{\phi(G_g)} = \ell(\phi(G_g))$ be the bit length of $\phi(G_g)$ and $n_{\phi(G_{Rg})} = \ell(\phi(G_{Rg}))$ be the bit length of $\phi(G_{Rg})$. Let q be a random k-bit prime. Let $Q = \prod_{i=1}^{25} p_i = 116431182179248680450031658440253681535$. The system parameters are $(g,k,q,Q,N,R,G_g,G_{Rg},n,n_{\phi(G_g)})$.

7.4 KAZ-SIGN Algorithms

The full algorithms of KAZ-SIGN are shown in Algorithms 1, 2, and 3.

Algorithm 1 KAZ-SIGN Key Generation Algorithm

```
Input: System parameters $(g,k,q,Q,N,R,G_g,G_{Rg},n,n_{\phi(G_g)})$
Output: Public verification key pair, $(V_1,V_2)$, private signing key, $\alpha$, and secret signing
    key, $SK$
1: Choose random prime $(a,\alpha)$ and random $\omega_1 \in (2^{n_{\phi(G_g)}-2}, 2^{n_{\phi(G_g)}-1})$.
2: Compute public verification key-1, $V_1 \equiv \alpha \pmod{G_{Rg}}$.
3: Compute secret parameter $b \equiv a^{\phi(\phi(G_{Rg}))} \pmod{\omega_1 \phi(G_{Rg})}$.
4: Compute public verification key-2, $V_2 \equiv Q(\alpha^{\phi(Q)b}) \pmod{qQ}$.
5: Compute secret signing key, $SK \equiv \alpha^{\phi(Q)b} \pmod{G_{Rg}qQ}$
6: Output public verification keys, $(V_1,V_2)$, keep signing key $SK$ secret and destroy pa-
    rameters $(\alpha,a,b,\omega_1)$.
```

Algorithm 2 KAZ-SIGN Signing Algorithm

```
Input: System parameters $(g,k,q,Q,N,R,G_g,G_{Rg},n,n_{\phi(G_g)})$, private signing key, $\alpha$, se-
    cret signing key, $SK$ and message to be signed, $m \in \mathbb{Z}_N$.
Output: Signature, $S$
1: Let $m \in \mathbb{Z}_N$ be the message to be signed and let $h = nextprime(H(m))$.
2: Choose random prime $r$, and random $\omega_2 \in (2^{n_{\phi(G_g)}-2}, 2^{n_{\phi(G_g)}-1})$.
3: Compute secret parameter $\beta \equiv r^{\phi(\phi(G_{Rg}))} \pmod{\omega_2 \phi(G_{Rg})}$.
4: Compute $S \equiv (SK)(h^{(\phi(qQ)\beta)}) \pmod{G_{Rg}qQ}$.
5: Output signature, $S$, and destroy $(\beta,r,\omega_2)$.
```

Algorithm 3 KAZ-SIGN Verification Algorithm

```
Input: System parameters $(g,k,q,Q,N,R,G_g,G_{Rg},n,n_{\phi(G_g)})$, public verification key pair,
    $(V_1,V_2)$, message, $m$, and signature, $S$.
Output: Accept or reject signature
 1: Compute $h = nextprime(H(m))$.
 2: Compute $y \equiv (V_1^{\phi(Q)})(h^{\phi(qQ)}) \pmod{G_{Rg}Q}$ and $S_{F1} = CRT([\frac{V_2}{Q}, y], [q, G_{Rg}Q])$.
 3: Compute the following procedure: Set $S_{F2} = 0$. Set $modulus = 1$ and $soln = 0$.
 4:     $VQ \equiv (V_1^{\phi(Q)})(h^{\phi(qQ)}) \pmod{G_{Rg}}$
 5:     for each factor $r_i^{e_i}$ of $G_{Rg}qQ$ do
 6:             set $g_1 = \gcd(Q, r_i^{e_i})$
 7:             set $g_2 = \gcd(G_{Rg}, r_i^{e_i})$
 8:             set $g_3 = \gcd(qQ, r_i^{e_i})$
 9:             if $g_1 \neq 1$ then $soln = 1$; end if
10:             if $g_1 = 1$ and $g_2 \neq 1$ and $(g_3 = 1$ or $Q \pmod{g_3} = 0)$ then
                $soln = VQ \pmod{g_2}$; end if
11:             if $g_1 = 1$ and $g_2 = 1$ then $soln = V_2Q^{-1}$; end if
12:             $S_{F2} = CRT([S_{F2}, soln], (modulus, r_i^{e_i}))$
13:             $modulus = modulus \cdot r_i^{e_i}$
14:     end for
15: Compute $w_0 \equiv (S \pmod{G_{Rg}qQ}) - S$.
16: if $w_0 \neq 0$ then
17:     Reject signature $\perp$
18: else Continue Step 20
19: end if
20: Compute $w_1 \equiv (S \pmod{G_{Rg}qQ}) - S_{F1}$.
21: if $w_1 = 0$ then
22:     Reject signature $\perp$
23: else Continue Step 25
24: end if
25: Compute $w_2 = (S \pmod{G_{Rg}qQ}) - S_{F2}$
26: if $w_2 = 0$ then
27:     Reject signature $\perp$
28: else Continue Step 30
29: end if
30: Compute $w_3 \equiv QS \pmod{qQ}$. Compute $w_4 = w_3 - V_2$.
31: if $w_4 \neq 0$ then
32:     Reject signature $\perp$
33: else Continue Step 35
34: end if
35: Compute $y_1 \equiv g^{(R^S \pmod{G_g})} \pmod{N}$.
36: Compute $y_2 \equiv g^{(R^{((V_1^{\phi(Q)})(h^{\phi(qQ)}) \pmod{G_{Rg}})} \pmod{G_g})} \pmod{N}$.
37: if $y_1 = y_2$ then
38:     accept signature
39: else reject signature $\perp$
40: end if
```

Steps 15, 16, 17, 18, and 19 during verification are known as the **KAZ-SIGN digital signature forgery detection procedure type – 1**, steps 20, 21, 22, 23, and 24 during verification are known as the **KAZ-SIGN digital signature forgery detection procedure type – 2**, steps 25, 26, 27, 28, and 29 during verification are known as the **KAZ-SIGN digital signature forgery detection procedure type – 3** and steps 30, 31, 32, 33, and 34 during verification are known as the **KAZ-SIGN digital signature forgery detection procedure type – 4**.

8. THE DESIGN RATIONALE

In this section we will analyse the rationale behind the design vis-à-vis a valid signature parameter S.

8.1 Proof of correctness (Verification steps 35, 36, 37, 38, 39, and 40)

We begin by discussing the rationale behind steps 35, 36, 37, 38, 39, and 40 with relation to the verification process. Observe the following,

$$\begin{aligned}
g^{(R^S \pmod{G_g})} &\equiv g^{R^{((\alpha^{(\phi(Q)b)})(h^{(\phi(qQ)\beta)}) \pmod{G_{Rg}})}} \pmod{G_g} \\
&\equiv g^{R^{((\alpha^{(\phi(Q))})(h^{(\phi(qQ))}) \pmod{G_{Rg}})}} \pmod{G_g} \\
&\equiv g^{(R^{((V_1^{\phi(Q)}(h^{\phi(qQ)}) \pmod{G_{Rg}})} \pmod{G_g})} \quad \pmod{N}
\end{aligned}$$

because $\alpha \equiv V_1 \pmod{G_{Rg}}$, and $b \equiv a^{\phi(\phi(G_{Rg}))} \equiv 1 \pmod{\phi(G_{Rg})}$ since $\omega_1 \phi(G_{Rg}) \equiv 0 \pmod{G_{Rg}}$. As such the verification process does indeed provide an indication that the signature is indeed from an authorized sender with the private signing key α.

8.2 Proof of correctness (Verification steps 15, 16, 17, 18, and 19: KAZ-SIGN digital signature forgery detection procedure type – 1)

In order to comprehend the rationale behind steps 15, 16, 17, 18, and 19, one has to observe the following,

$$w_0 \equiv (S \pmod{G_{Rg}qQ}) - S = 0$$

because $S < G_{Rg}qQ$.

8.3 Proof of correctness (Verification steps 20, 21, 22, 23, and 24: KAZ-SIGN digital signature forgery detection procedure type – 2)

In order to comprehend the rationale behind steps 20, 21, 22, 23, and 24, one has to observe the following; obviously S_{F1} is not constructed with secret parameters (α, b). As such from $w_1 \equiv (S \pmod{G_{Rg}qQ}) - S_{F1}$, we will have $w_1 \neq 0$.

8.4 Proof of correctness (Verification steps 25, 26, 27, 28, and 29: KAZ-SIGN digital signature forgery detection procedure type – 3)

In order to comprehend the rationale behind steps 25, 26, 27, 28, and 29, one has to observe the following; obviously S_{F2} is not constructed with secret parameters (α, b). As such from $w_2 \equiv (S \pmod{G_{Rg}qQ}) - S_{F2}$, we will have $w_2 \neq 0$.

8.5 Proof of correctness (Verification steps 30, 31, 32, 33, and 34: KAZ-SIGN digital signature forgery detection procedure type – 4)

In order to comprehend the rationale behind steps 30, 31, 32, 33, and 34, one has to observe the following;

$$w_3 \equiv QS \equiv Q(\alpha^{\phi(Q)b}) \pmod{qQ}$$

Hence, $w_4 = w_3 - V_2 = 0$.

8.6 Deriving forged signature identifiable by KAZ-SIGN digital signature forgery detection procedure type – 1.

An adversary utilizing a valid signature, S and resends it as follows:

$$S_{F0} \equiv S + G_{Rg}qQx \pmod{\theta G_{Rg}qQ}$$

for some random value of $x \in \mathbb{Z}$ and small value of $\theta \in \mathbb{Z}$, such that $\ell(S_{F0}) \approx \ell(S)$. That is, $\ell(S_{F0})$ is not suspicious to the verifier. It is easy to observe that S_{F0} will pass steps 35, 36, 37, 38, 39, and 40. However, since

$$w_0 \equiv (S_{F0} \pmod{G_{Rg}qQ}) - S_{F0} \neq 0 \in \mathbb{Z}$$

the signature will fail KAZ-SIGN digital signature forgery detection procedure type – 1.

8.7 Deriving forged signature identifiable by KAZ-SIGN digital signature forgery detection procedure type – 2

An adversary that constructs a forged signature S as follows; compute $y \equiv (V_1^{\phi(Q)})(h^{\phi(qQ)}) \pmod{G_{Rg}Q}$ and $S = CRT([\frac{V_2}{Q}, y], [q, G_{Rg}Q])$, and then transmits it as a signature S would result in

$$w_1 \equiv (S \pmod{G_{Rg}qQ}) - S_{F1} = 0.$$

It is easy to observe that S will pass steps 35, 36, 37, 38, 39, and 40. However, the signature will fail KAZ-SIGN digital signature forgery detection procedure type - 2.

8.8 Deriving forged signature identifiable by KAZ-SIGN digital signature forgery detection procedure type - 3

An adversary that constructs a forged signature S as described in steps 3-14 within the verification algorithm; and then transmits it as a signature S would result in

$$w_2 \equiv S \pmod{G_{Rg}qQ} - S_{F2} = 0.$$

It is easy to observe that S will pass steps 35, 36, 37, 38, 39, and 40. However, the signature will fail KAZ-SIGN digital signature forgery detection procedure type - 3.

8.8.1 Origins of KAZ-SIGN digital signature forgery detection procedure type - 3

The origins of KAZ-SIGN digital signature forgery detection procedure type - 3 is from the following iterative CRT procedure.

```
1:  Compute the following procedure: Set $S_{F2} = 0$. Set $modulus = 1$.
2:      $VQ \equiv (V_1^{\phi(Q)})(h^{\phi(qQ)}) \pmod{G_{Rg}}$
3:      for each factor $r_i^{e_i}$ of $G_{Rg}qQ$ do
4:          for $soln = 0, 1, 2, \ldots, r_i^{e_i} - 1$ do
5:              if $soln$ mod $\gcd(Q, r_i^{e_i}) \not\equiv 1 \pmod{\gcd(Q, r_i^{e_i})}$ then next; end if
6:              if $soln$ mod $\gcd(G_{Rg}, r_i^{e_i}) \not\equiv VQ$ mod $\gcd(G_{Rg}, r_i^{e_i})$ then next; end if
7:              if $soln \cdot Q$ mod $\gcd(q \cdot Q, r_i^{e_i}) \not\equiv V_2$ mod $\gcd(q \cdot Q, r_i^{e_i})$then next; end if
8:              break
9:          end for
10:         $S_{F2} = CRT([S_{F2}, soln], (modulus, r_i^{e_i}))$
11:         $modulus = modulus \cdot r_i^{e_i}$
12:     end for
```

For large q, the iterative CRT would not be feasible. Nevertheless, if the above iterative CRT could be enhanced, it would produce S_{F2} that would pass all verification procedures. KAZ-SIGN digital signature forgery detection procedure type - 3, is an enhanced implementation of the above iterative CRT procedure.

8.9 Deriving forged signature identifiable by KAZ-SIGN digital signature forgery detection procedure type – 4

An adversary that constructs a forged signature S without the private key α and at the same time aspires to pass steps 35, 36, 37, 38, 39, and 40 would result in the relation,

$$S \equiv (\lambda^{\phi(Q)b}) \pmod{qQ}$$

where $\lambda = V_1 + G_{Rg}t$ for some $t \in \mathbb{Z}$. It is clear that $\alpha \not\equiv V_1 + G_{Rg}t \pmod{qQ}$. As such, $w_4 = w_3 - V_2 \neq 0$, where $w_3 \equiv Q(\lambda^{\phi(Q)b}) \pmod{qQ}$. Thus, the signature will fail KAZ-SIGN digital signature forgery detection procedure type - 4.

8.10 Extracting α

An approach to forge the signature would be to produce either one of the following:

1. $y_{\alpha 1} \equiv \alpha \pmod{G_{Rg}qQ}$ OR
2. $y_{\alpha 2} \equiv \alpha^{\phi(Q)} \pmod{G_{Rg}qQ}$.

8.10.1 Producing $y_{\alpha 1} \equiv \alpha \pmod{G_{Rg}qQ}$

From the public parameter $V_1 \equiv \alpha \pmod{G_{Rg}}$, the adversary needs to obtain the parameter $\alpha \pmod{qQ}$ to execute the Chinese Remainder Theorem (CRT) to obtain $\alpha \pmod{G_{Rg}qQ}$. To obtain $\alpha \pmod{qQ}$, the adversary will utilize equation S. Observe that

$$S \equiv (\alpha^{\phi(Q)b})(h^{\phi(qQ)\beta}) \equiv \alpha^{\phi(Q)b} \not\equiv \alpha \pmod{qQ}$$

Thus, this option is not viable.

8.10.2 Producing $y_{\alpha 2} \equiv \alpha^{\phi(Q)} \pmod{G_{Rg}qQ}$

To obtain $y_{\alpha 2}$, one begins with,

$$z_1 \equiv V_1^{\phi(Q)} \equiv \alpha^{\phi(Q)} \pmod{G_{Rg}}.$$

Then, one needs to produce the parameter $\alpha^{\phi(Q)} \pmod{qQ}$. However,

$$z_2 \equiv S \equiv (\alpha^{(\phi(Q)b)})(h^{(\phi(qQ)\beta)}) \not\equiv \alpha^{\phi(Q)} \pmod{qQ}.$$

Thus, with the available parameters (S, V_1), one is unable to produce $y_{\alpha 2}$.

8.11 Modular linear equation of S

In this direction we analyze

$$S \equiv (\alpha^{(\phi(Q)b)})(h^{(\phi(qQ)\beta)}) \pmod{G_{Rg}qQ}$$

Let

1. $X_1 \equiv \alpha^{\phi(Q)b} \pmod{G_{Rg}qQ}$
2. $X_2 \equiv h^{\phi(qQ)\beta} \pmod{G_{Rg}qQ}$

Moving forward we have,

$$X_1X_2 - S \equiv 0 \pmod{G_{Rg}qQ} \tag{1}$$

Let $\hat{X_1}$ be the upper bound for X_1 and $\hat{X_2}$ be the upper bound for X_2. From Conjecture 1, if one has the situation where $\hat{X_1}\hat{X_2} \gg G_{Rg}qQ$, then there is no efficient algorithm to output all the roots of (1). That is, (1) usually has $G_{Rg}qQ$ many solutions, which is exponential in the bit-size of $G_{Rg}qQ$.
To this end, since both $\alpha^{\phi(Q)b}$ and $h^{\phi(qQ)\beta}$ are exponentially large, it is clear to conclude that $\hat{X_1}\hat{X_2} \gg G_{Rg}qQ$. This implies, there is no efficient algorithm to output all the roots of (1).

8.12 Implementation of the Hidden Number Problem (HNP)

From S, let us denote as follows:

1. $x_1 \equiv \alpha^{(\phi(Q)b)} \pmod{G_{Rg}qQ}$
2. $x_2 \equiv \phi(qQ)\beta$

Thus, S can be re-written as

$$S \equiv (x_1)(h^{x_2}) \pmod{G_{Rg}qQ} \tag{2}$$

for unknown pair (x_1, x_2). It is obvious that (2) is the HNP.

8.13 Analysis on V_2

Assume we have $V_1 \equiv \alpha \pmod{q}$. Let,

$$w_1 \equiv V_1^{\phi(Q)} \equiv \alpha^{\phi(Q)} \pmod{q}$$

$$w_2 \equiv V_2Q^{-1} \equiv \alpha^{\phi(Q)b} \equiv V_1^{\phi(Q)b} \pmod{q}$$

The aim is to obtain the system of equations

$$b \equiv \zeta \pmod{\phi(q)} \tag{3}$$

$$b \equiv a^{\phi(\phi(G_{Rg}))} \pmod{\phi(G_g)} \tag{4}$$

for some arbitarily chosen prime a.

We obtain ζ by solving the DLP upon $w_2 \equiv \alpha^{\phi(Q)b} \equiv V_1^{\phi(Q)b} \pmod{q}$, where the DL solver works on the base given by $w_1 \equiv V_1^{\phi(Q)} \pmod{q}$. That is, $z_1 \equiv b \equiv \zeta \pmod{\phi(q)}$

and $w_1^{z_1} \equiv V_1^{\phi(Q)b} \equiv w_2 \pmod{q}$. Then, for some prime a, let $z_2 \equiv a^{\phi(\phi(G_{Rg}))} \pmod{\phi(G_g)}$.

Let $z_3 = \gcd(\phi(q), \phi(G_g))$. Solving the CRT upon (3) and (4) modulo $(\frac{\phi(q)\phi(G_g)}{z_3})$ would result in $z_4 \pmod{\frac{\phi(q)\phi(G_g)}{z_3}}$, and since $\frac{\phi(q)\phi(G_g)}{z_3} \equiv 0 \pmod{\phi(q)}$, and if $\phi(Q) \equiv 0 \pmod{z_3}$, we have

$$z_4 \equiv \zeta \pmod{\frac{\phi(q)}{z_3}} \Rightarrow \phi(Q)z_4 \equiv \phi(Q)\zeta \pmod{\phi(q)}$$
$$z_4 \equiv a^{\phi(\phi(G_{Rg}))} \pmod{\phi(G_g)}$$

Hence, we have
$$Q(V_1^{\phi(Q)z_4}) \equiv Q(V_1^{\phi(Q)\zeta}) \equiv V_2 \pmod{qQ}.$$

As such, forgery is doable. That is, the forged signature is of the form

$$S^* \equiv (V_1^{\phi(Q)z_4})(h^{\phi(qQ)}) \pmod{G_{Rg}qQ}$$

That is,
$$QS^* \equiv Q(V_1^{\phi(Q)z_4}) \equiv V_2 \pmod{qQ}$$

And,

$$\begin{aligned} y_1 &\equiv g^{R^{S^*}} \ (\mathrm{mod}\ G_g) \\ &\equiv g^{R^{(V_1^{\phi(Q)z_4})(h^{\phi(qQ)})\ (\mathrm{mod}\ G_{Rg})}\ (\mathrm{mod}\ G_g)} \pmod{N} \\ &\equiv g^{R^{(V_1^{\phi(Q)(1)})(h^{\phi(qQ)})\ (\mathrm{mod}\ G_{Rg})}\ (\mathrm{mod}\ G_g)} \pmod{N} \\ y_2 &\equiv g^{R^{(V_1^{\phi(Q)})(h^{\phi(qQ)})\ (\mathrm{mod}\ G_{Rg})}\ (\mathrm{mod}\ G_g)} \pmod{N} \end{aligned}$$

We have $y_1 = y_2$. As such forgery can occur.

Furthermore, when $\phi(Q) \not\equiv 0 \pmod{z_3}$, we have

$$z_4 \equiv \zeta \pmod{\frac{\phi(q)}{z_3}} \Rightarrow \phi(Q)z_4 \not\equiv \phi(Q)\zeta \pmod{\phi(q)}$$

$$z_4 \equiv a^{\phi(\phi(G_{Rg}))} \pmod{\phi(G_g)}$$

Hence, we have
$$Q(V_1^{\phi(Q)z_4}) \not\equiv Q(V_1^{\phi(Q)\zeta}) \equiv V_2 \pmod{qQ}$$

Thus, to satisfy the filtering process of $QS \equiv V_2 \pmod{qQ}$,

we utilize

$$Q(w_1^{z_1}) \equiv Q(V_1^{\phi(Q)b}) \equiv V_2 \pmod{qQ} \tag{5}$$

In order to ensure (5) is executable, the signature is of the form

$$S^* \equiv (w_1^{z_1})(h^{\phi(qQ)\beta}) \pmod{G_{Rg}qQ}$$

That is,

$$QS^* \equiv Q(w_1^{z_1}) \equiv V_2 \pmod{qQ}$$

And

$$\begin{aligned} y_1 &\equiv g^{R^{S^*} \text{ (mod } G_g)} \\ &\equiv g^{R^{(w_1^{z_1})(h^{\phi(qQ)}) \text{ (mod } G_{Rg})} \text{ (mod } G_g)} \pmod{N} \\ &\equiv g^{R^{(V_1^{\phi(Q)b})(h^{\phi(qQ)}) \text{ (mod } G_{Rg})} \text{ (mod } G_g)} \pmod{N} \\ y_2 &\equiv g^{R^{(V_1^{\phi(Q)})(h^{\phi(qQ)}) \text{ (mod } G_{Rg})} \text{ (mod } G_g)} \pmod{N} \end{aligned}$$

We have $y_1 = y_2$. As such forgery can occur.

However, the value $V_1 \equiv \alpha \pmod{q}$ is not available.

9. SPECIFICATION OF KAZ-SIGN

The challenge faced by the adversary is to retrieve α from $V_1 \equiv \alpha \pmod{G_{Rg}}$. It is protected by the MRP. The MRP representation is given as follows:

$$t = \frac{\alpha - V_1}{G_{Rg}}$$

Due to the strategies during key generation, we have the complexity $O(t) = O(q)$.

As such, the complexity of solving the MRP via $V_1 \equiv \alpha \pmod{G_{Rg}}$ will be the determining factor in identifying the suitable key length for each security level.

The following is the security specification for $\delta = 0.274$.

Number of primes in P	$\ell(q)$	$n = \ell(N)$	Total security level, k
128	128	999	128
192	192	1631	192
256	256	2300	256

Table 1

10. IMPLEMENTATION AND PERFORMANCE

10.1 Key Generation, Signing and Verification Time Complexity

It is obvious that the time complexity for all three procedures is in polynomial time.

10.2 Parameter sizes

We provide here information on size of the key and signature based on security level information from Table 1 (for $\delta = 0.274$).

NIST Security Level	Number of primes in P	Security level, k	Length of parameter N (bits)	Public key size, (V_1, V_2) (bits)	Private Signing key Size, SK	Signature Size (S) (bits)	ECC key size (bits)
1	128	128	999	≈ 78 $+252$ $= 330$	≈ 333	≈ 333	256
3	192	192	1631	≈ 90 $+315$ 405	≈ 406	≈ 406	384
5	256	256	2300	≈ 140 $+382$ $= 522$	≈ 522	≈ 522	521

Table 2

In the direction of the research, we also make comparison to ECC key length for the three NIST security levels. KAZ-SIGN key length did not achieve its immediate target of having approximately the same key length as ECC, but further research might find means and ways.

10.3 Key Generation, Signing and Verification Ease of Implementation

The algebraic structure of KAZ-SIGN has an abundance of programming libraries available to be utilized. Among them are:

1. GNU Multiple Precision Arithmetic Library (GMP); and
2. Standard C libraries.

10.4 Key Generation, Signing and Verification Empirical Performance Data

In order to obtain benchmarks, we evaluate our reference implementation on a machine using GCC Compiler Version 6.3.0 (MinGW.org GCC-6.3.0-1) on Windows 10 Pro, Intel(R) Core(TM) i7-4710HQ CPU @ 2.50GHz and 8.00 GB RAM (64-bit operating system, x64-based processor).

We have the following empirical results when conducting 100 key generations, 100 signings and 100 verifications:

Security level	Time (ms)		
	Key generation	Signing	Verification
128 - KAZ999	256	236	127
192 - KAZ1631	332	345	256
256 - KAZ2300	571	675	644

Table 3

11. ADVANTAGES AND LIMITATIONS

As we have seen, KAZ-SIGN can be evaluated through:

1. Key length
2. Speed
3. No verification failure

11.1 Key Length

KAZ-SIGN key length is comparable to non-post quantum algorithms such as ECC and RSA. For 256-bit security, the KAZ-SIGN key size is approximate 522-bits. ECC would use 521-bit keys and RSA would use 15360-bit keys.

11.2 Speed

KAZ-SIGN's speed analysis results stem from the fact that it has short key length to achieve 256-bit security plus its textbook complexity running time for both signing and verifying is $O(n^3)$ where parameter n here is the input length.

11.3 No verification failure

It is apparent that the execution of **KAZ-SIGN parameter suitability detection procedure** together with **KAZ-SIGN digital signature forgery detection procedure type – 1, type – 2, type – 3, and type – 4** within the verification procedure will enable the verification computational process by the recipient to verify or reject a digital signature that was received by the recipient with probability equal to 1. That is, the probability of verification failure is 0.

11.4 Limitation

As we have seen, limitation of KAZ-SIGN can be evaluated through:

1. Based on unknown problem, the Modular Reduction Problem (MRP)

11.4.1 Based on unknown problem, the Modular Reduction Problem (MRP)

The MRP is not a known hard mathematical problem which is quantum resistant and is subject to future cryptanalysis success in solving the defined challenge either with a classical or quantum computer.

12. CLOSING REMARKS

The KAZ-SIGN digital signature exhibits properties that might result in it being a desirable post quantum signature scheme. In the event that new forgery methodologies are found, as long as the procedure can also be done by the verifier, then one can add the new forgery methodology into the verification procedure. At the same time, the same forgery methodology can be inserted into the signing procedure in order to eliminate any chances the signer will produce a signature that will be rejected.

To this end, the security of the MRP is an unknown fact. We opine that, the acceptance of MRP as a potential quantum resistant hard mathematical problem will come hand in hand with a secure cryptosystem designed upon it. We welcome all comments on the KAZ-SIGN digital signature, either findings that nullify its suitability as a post quantum digital signature scheme or findings that could enhance its deployment and use case in the future.

Finally, we would like to put forward our heartfelt thanks to Prof. Dr. Abderrahmane Nitaj from Laboratoire de Mathématiques Nicolas Oresme, Université de Caen Basse Normandie, France for insights, comments, and friendship throughout the process. Next, special thanks to Prof. Dr. Daniel J. Bernstein from University of Illinois at Chicago, United States of America who has given his thoughts and efforts throughout versions 1.0 until 1.5β.2 of KAZ-SIGN. Today, our participation in this NIST exercise has lead us towards new collaborations.

13. ILLUSTRATIVE FULL SIZE TEST VECTORS – 1

The following are parameters that illustrate KAZ-SIGN for 128-bit security (refer to Table 2). This is an example for $j = 128$. That is, $P = \{3, 5, 7, \ldots, 727\}$. In this illustration, we provide a valid KAZ-SIGN signature S. The valid KAZ-SIGN signature will pass all 4 KAZ-SIGN digital signature forgery detection procedure types.

N :

36074125839713232195900037838349442849885202263194537439069698009198951765514300
71528816513023400139449183391549534086592248781036129317137092037483563399346623
61455775104479726899100647924875923316015883645117612153454073013122198477918174
3430655484717223193007705468625920195527456360287632608176655 $\approx 2^{999}$

g :

6007

G_g :

23102151283542472555351033031857407110549489214984451103786304558150674606117088
000 $\approx 2^{274} \approx 2^{0.274(999)} \approx N^{0.274}$

R :

6151

G_{Rg} :

399620650696124709852000 $\approx 2^{79} \approx N^{0.079}$

q :

174191248036859359750069958033351987111

Q :

116431182179248680450031658440253681535

Key generation

α :
196476610189668651834375539590674880823165744286922742833787387 $\approx 2^{207}$

V_1 :
194899524029958783335387

a :
9557604543589816246292689859129704969819285176879070695350406126687660468882968887

ω_1 :
13213552022081672541383421172567197642291569021582977338374809877470026814247362019

b :
712849563102150043508184687794612951740754028005598774986063735200998831010006747057132535511043276800001

V_2 :
8692183052973937452543725608081522880099206692068552531594512329261446225040

SK :
6304973040999257983498216162170756025176406615498770229999620623620887366332263823517539001212100001

MRP complexity upon $t_{\alpha V_1}$

$t = \dfrac{\alpha - V_1}{G_{Rg} q}$:
491657800585163724786384201761939974051 $\approx 2^{129}$

Signing

h :
7329593292295158914231822146100984721636601876217417946484086058812307337 3191

r :
94644440311313516436535538137632241261301771559525281528486492570923900332364853
03

ω_2 :
11924278146000559293571249563331189924910399489207325712060794519232567790806805
899

β :
28964322149342398929264477382598573723798429286078545791104134236740669918623491
88233987637161387622400 01

S :
39381322487997741534934892188135583160854042132229726018647407678729332840911129
69314978996891740001

Verification

KAZ-SIGN digital signature forgery detection procedure type – 1

w_0 :
0

KAZ-SIGN digital signature forgery detection procedure type – 2

S_{F1} :
10097269070589287716103759197732633690327277981981303529624590815555334308905802
76752628166026700001

w_1 :
29284053417408453818831132990402949470526764150248422489022816863173998532005326
92562350830865040000 $\neq 0$

KAZ-SIGN digital signature forgery detection procedure type – 3

S_{F2} :
53611781495237774051255701682438186740748915596555741877636108783127397420219766
66507318635001880001

w_2 :
$-$ 14230459007240032516320809494302603579894873464326015858988701104398064579308636
971923396381101400000 $\neq 0$

KAZ-SIGN digital signature forgery detection procedure type – 4

w_3 :
8692183052973937452543725608081522880099206692068552531594512329261446225040

w_4 :
0

Final verification

y_1 and y_2 :
56716223308104667361111018920155488582517110296610738663192091826146613921171515
17993753555469587188851902600395414019752909596162954105825796772087989588585483
46985270650512513472176263500070397543969892711780071611975313265422659206968332
244331695704845804441296648765120082101079175606771307856608

14. ILLUSTRATIVE FULL SIZE TEST VECTORS – 2

The following are parameters that illustrate KAZ-SIGN for 128-bit security (refer to Table 2). This is an example for $j = 128$. That is, $P = \{3, 5, 7, \ldots, 727\}$. In this illustration, we provide a forged KAZ-SIGN signature S where the system parameters, $(N, g, q, Q, G_g, R, G_{Rg}, \alpha, V_1, V_2, h)$ are the same as in **ILLUSTRATIVE FULL SIZE TEST VECTORS – 1** and the forged signature is of the form of $S \equiv S_V + G_{Rg}qQ$ where S_V is a valid signature. This signature will fail the **KAZ-SIGN digital signature forgery detection procedure type – 1**.

S_V :
3938132248799774153493489218813558316085404213222972601864740767872933284091112969314978996891740001

S :
1204295572812759712027467081530782668232273062046798651066916012209268456824913788680184889332476000 1

KAZ-SIGN digital signature forgery detection procedure type – 1

w_0 :
$-8104823479327822966781181596494268366237326407245013908804419354219751284158024917486869896433020000 \neq 0$

Final verification

y_1 and y_2 :
5671622330810466736111101892015548858251711029661073866319209182614661392117151517993753555469587188851902600395414019752909596162954105825796772087989588585483469852706505125134721762635000703975439698927117800716119753132654226592069683322443316957048458044412966487651200821010791756067713078566 08

15. ILLUSTRATIVE FULL SIZE TEST VECTORS – 3

The following are parameters that illustrate KAZ-SIGN for 128-bit security (refer to Table 2). This is an example for $j = 128$. That is, $P = \{3, 5, 7, \ldots, 727\}$. In this illustration, we provide a forged KAZ-SIGN signature S where the system parameters, $(N, g, q, Q, G_g, R, G_{Rg}, \alpha, V_1, V_2, h)$ are the same as in **ILLUSTRATIVE FULL SIZE TEST VECTORS – 1** and the forged signature is of the form of $S \equiv (S_V \pmod{\frac{G_{Rg}qQ}{e}}) + G_{Rg}qQ$ where S_V is a valid signature and e is an n integer consisting some or all common primes between G_{Rg} and Q. This signature will fail the <u>**KAZ-SIGN digital signature forgery detection procedure type – 1**</u>.

e :
103357296372885555

$S_V \pmod{\dfrac{G_{Rg}qQ}{e}}$:
797543355236349089314704035502888321520487510930503725635770579717322992985977 20
01

S :
8104823479327822974756615148857759259384366762273897124009294463524788540515730 7
14660099826292792001

<u>**KAZ-SIGN digital signature forgery detection procedure type – 1**</u>

w_0 :
$-$8104823479327822966781181596494268366237326407245013908804419354219751284158024
917486869896433020000 $\neq 0$

<u>**Final verification**</u>

y_1 and y_2 :
567162233081046673611110189201554885825171102966107386631920918261466139211715 15
179937535554695871888519026003954140197529095961629541058257967720879895885854 83
469852706505125134721762635000703975439698927117800716119753132654226592069683 32
2443316957048458044412966487651200821010791756067713078 56608

16. ILLUSTRATIVE FULL SIZE TEST VECTORS – 4

The following are parameters that illustrate KAZ-SIGN for 128-bit security (refer to Table 2). This is an example for $j = 128$. That is, $P = \{3, 5, 7, \ldots, 727\}$. In this illustration, we provide a forged KAZ-SIGN signature S where the system parameters, $(N, g, q, Q, G_g, R, G_{Rg}, \alpha, V_1, V_2, h)$ are the same as in **ILLUSTRATIVE FULL SIZE TEST VECTORS – 1** and conduct the CRT upon the equation pair $Y_1 = V_2 Q^{-1}$ and $Y_2 \equiv (V_1^{\phi(Q)})(h^{\phi(qQ)}) \pmod{G_{Rg}Q}$ to obtain a forge signature. This signature will fail the <u>**KAZ-SIGN digital signature forgery detection procedure type – 2**</u>.

Y_1 :
74655112919768408542199014975763442544

Y_2 :
14914695613722113833275117852910196235999012705859528597800001

S :
10097269070589287716103759197732633690327277981981303529624590815555334308905802
76752628166026700001

<u>**KAZ-SIGN digital signature forgery detection procedure type – 2**</u>

S_{F1} :
10097269070589287716103759197732633690327277981981303529624590815555334308905802
76752628166026700001

w_1 :
0

<u>**Final verification**</u>

y_1 and y_2 :
56716223308104667361111018920155488582517110296610738663192091826146613921171515
17993753555469587188851902600395414019752909596162954105825796772087989588585483
46985270650512513472176263500070397543969892711780071611975313265422659206968332
24433169570484580444129664876512008210107917560677130785660 8

17. ILLUSTRATIVE FULL SIZE TEST VECTORS – 5

The following are parameters that illustrate KAZ-SIGN for 128-bit security (refer to Table 2). This is an example for $j = 128$. That is, $P = \{3, 5, 7, \ldots, 727\}$. In this illustration, we provide a forged KAZ-SIGN signature S where the system parameters, $(N, g, q, Q, G_g, R, G_{Rg}, \alpha, V_1, V_2, h)$ are the same as in **ILLUSTRATIVE FULL SIZE TEST VECTORS – 1** and the forge signature is constructed as per steps 3 – 14 and 25 – 29 during verification. This signature will fail the **KAZ-SIGN digital signature forgery detection procedure type – 3**.

VQ :
6697552804962983964001

S :
536117814952377740512557016824381867407489155965557418776361087831273974202197666
6507318635001880001

KAZ-SIGN digital signature forgery detection procedure type – 3

S_{F2} :
536117814952377740512557016824381867407489155965557418776361087831273974202197666
6507318635001880001

w_2 :
0

Final verification

y_1 and y_2 :
567162233081046673611110189201554885825171102966107386631920918261466139211715 15
179937535554695871888519026003954140197529095961629541058257967720879895885854 83
469852706505125134721762635000703975439698927117800716119753132654226592069683 32
2443316957048458044412966487651200821010791756067713078566 08

18. ILLUSTRATIVE FULL SIZE TEST VECTORS – 6

The following are parameters that illustrate KAZ-SIGN for 128-bit security (refer to Table 2). This is an example for $j = 128$. That is, $P = \{3, 5, 7, \ldots, 727\}$. In this illustration, we provide a forged KAZ-SIGN signature S where the system parameters, $(N, g, q, Q, G_g, R, G_{Rg}, \alpha, V_1, V_2, h)$ are the same as in **ILLUSTRATIVE FULL SIZE TEST VECTORS – 1** and $S \equiv (V_1^{(\phi(Q))})(h^{(\phi(qQ))}) \pmod{G_{Rg}qQ}$. This signature will fail the **KAZ-SIGN digital signature forgery detection procedure type – 4**.

S :
38042996445964830648755454405525963151259115544886372613476250685811390913223238
89683076695993960001

KAZ-SIGN digital signature forgery detection procedure type – 4

w_3 :
18613802110769639964300857169960801301058746292795838429205226770210083473670

w_4 :
9921619057795702511757131561879278420959539600727285897610714440948637248630 $\neq 0$

Final verification

y_1 and y_2 :
56716223308104667361111018920155488582517110296610738663192091826146613921171515
17993753555469587188851902600395414019752909596162954105825796772087989588585483
46985270650512513472176263500070397543969892711780071611975313265422659206968332
244331695704845804441296648765120082101079175606771307856608

19. ILLUSTRATIVE FULL SIZE TEST VECTORS – 7

The following are parameters that illustrate KAZ-SIGN for 128-bit security (refer to Table 2). This is an example for $j = 128$. That is, $P = \{3, 5, 7, \ldots, 727\}$. In this illustration, we provide a forged KAZ-SIGN signature S where the system parameters, $(N, g, q, Q, G_g, R, G_{Rg}, \alpha, V_1, V_2, h)$ are the same as in **ILLUSTRATIVE FULL SIZE TEST VECTORS – 1** and conduct the CRT upon the equation pair $Y_1 \equiv 1 \pmod{\frac{qQ}{w}}$ where $w = \gcd(Q, G_{Rg})$ and $Y_2 \equiv (V_1^{\phi(Q)})(h^{\phi(qQ)}) \pmod{G_{Rg}}$ to obtain a forge signature. This signature will fail the **KAZ-SIGN digital signature forgery detection procedure type – 4**.

Y_1 :
1

Y_2 :
66975528049629839640 01

S :
26284555642736504969299540591844723090432816234658316749546780401442801829218960
01

KAZ-SIGN digital signature forgery detection procedure type – 4

w_3 :
116431182179248680450031658440253681535

w_4 :
$-8692183052973937452543725608081522879982775509889303851144480670821192543505 \neq 0$

Final verification

y_1 and y_2 :
56716223308104667361111018920155488582517110296610738663192091826146613921171515
17993753555469587188851902600395414019752909596162954105825796772087989588585483
46985270650512513472176263500070397543969892711780071611975313265422659206968332
244331695704845804441296648765120082101079175606771307856608

20. ILLUSTRATIVE FULL SIZE TEST VECTORS – 8

The following are parameters that illustrate KAZ-SIGN for 128-bit security (refer to Table 2). This is an example for $j = 128$. That is, $P = \{3, 5, 7, \ldots, 727\}$. In this illustration, we provide a forged KAZ-SIGN signature S where the system parameters, $(N, g, q, Q, G_g, R, G_{Rg}, \alpha, V_1, V_2, h)$ are the same as in **ILLUSTRATIVE FULL SIZE TEST VECTORS – 1** and construct a forged signature with a forged α of the form $A = V_1 + G_{Rg}T$ for some $T \in \mathbb{Z}$ and forged α is a prime. The constructed forged signature is of the form $S \equiv (A^{\phi(Q)})(h^{\phi(qQ)}) \pmod{G_{Rg}qQ}$. This signature will fail the **KAZ-SIGN digital signature forgery detection procedure type – 4**.

T :
213352556447705824662451401171525808097

A :
14851561038089076109643553600947543748954979592453719290296043364531628512362307
269516045372563819387

S :
80880634017006938815494381502003284741231509940440612397389016057214202072251868
67660812680939440001

KAZ-SIGN digital signature forgery detection procedure type – 4

w_3 :
18613802110769639964300857169960801301058746292795838429205226770210083473670

w_4 :
9921619057795702511757131561879278420959539600727285897610714440948637248630 $\neq 0$

Final verification

y_1 and y_2 :
56716223308104667361111018920155488582517110296610738663192091826146613921171515
17993753555469587188851902600395414019752909596162954105825796772087989588585483
46985270650512513472176263500070397543969892711780071611975313265422659206968332
244331695704845804441296648765120082101079175606771307856608

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